SOGRO: New Low-Frequency Terrestrial Gravitational Wave Detector

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Introduction.—Terrestrial GW detectors are mostly based on Michelson-type laser interferometers with arm-length of a few km covering 10 to a few kHz with strain sensitivity up to a few times 10^{-23} Hz$^{-1/2}$ [1-3]. These detectors are optimized for the detection of compact binary coalescence events that typically produce strong signals at $f \sim 1$ kHz. Several astrophysical processes generate GWs below 10 Hz to which the advanced detectors will not be sensitive. However, seismic and Newtonian noises (NN) have been regarded as serious obstacles in constructing terrestrial GW detectors.

Due to the transverse nature of GWs, a detector that measures all the components of the curvature tensor could distinguish GWs from near-field Newtonian gravity. The tensor detector is also capable of resolving the source direction and polarization [4]. By combining six magnetically levitated superconducting test masses (TMs), one could construct a full-tensor detector that could reach sensitivity $\leq 10^{-20}$ Hz$^{-1/2}$ at 0.1-10 Hz. We name this detector SOGRO (Superconducting Omni-directional Gravitational Radiation Observatory). Details on the design and sensitivity of SOGRO, and the Newtonian noise mitigation in a tensor detector, have been published elsewhere [5, 6]. In this paper, we summarize the published work on SOGRO and extend it to a larger baseline Advanced SOGRO (aSOGRO). We show that aSOGRO would be sensitive enough to detect not only IMBH binaries but also the low-frequency precursor of stellar mass BH binaries like GW150914.

Design.— Figure 1 shows the TM configuration of SOGRO. Six superconducting TMs, each with three linear degrees of freedom, are levitated over three orthogonal mounting tubes. The TMs are made of niobium (Nb) in the shape of a rectangular shell. Superconducting levitation coils and sensing capacitors (not shown) are located in the gap between the TMs and the mounting tubes, as well as on the outer surfaces of the TMs. The along-axis motions of the two TMs on each coordinate axis are differenced to measure a diagonal component of the wave $h_{ii}(t)$. The cross-axis (rotational) motions of the four TMs on each coordinate plane are differenced to measure an off-diagonal component $h_{ij}(t)$, $i \neq j$.

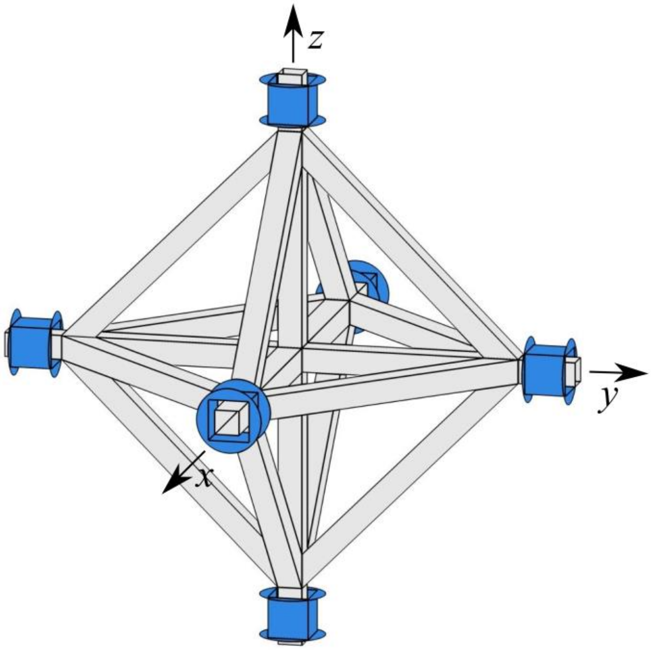

Figure 1. Test mass configuration for SOGRO. Six magnetically levitated test masses are combined to measure all six components of the curvature tensor.

Since TM motion is measured with respect to the sensing circuit elements mounted on the platform, SOGRO requires a *rigid platform* with mode frequencies above the signal bandwidth. To reduce its thermal noise, the platform itself needs to be cooled to ≤ 4.2 K. To alleviate excessive demand on cryogenics, the platform must not be too heavy while being rigid enough. Our preliminary FEM analysis shows that, by building the 3D cross with aluminum (Al) square tubes triangulated by Al circular tube struts (Fig. 1), a 30-m platform for SOGRO can be constructed with all the mode frequencies above 19 Hz and a total mass of ~70 tons. A 100-m platform for aSOGRO requires more sophisticated geometry with heavier mass. The magnetic levitation allows the TMs to have resonance frequencies ≤ 0.01 Hz with very high Q.

An extremely low-noise transducer with high energy coupling is required for a sensitive GW detector. For SOGRO, we propose to employ a *superconducting tuned capacitor bridge transducer* [7]. Figure 2 shows the transducer for a diagonal-component channel, where $C_{ij}(t)$ is a Nb capacitor plate located near the j-th face of the i-th TM. The bridge output is coupled to a near quantum-limited dc SQUID through a superconducting transformer. The sensing capacitances are modulated by GWs at ω and the bridge is driven at the resonance frequency of the capacitor bridge coupled to the SQUID input coil ω_p (>> ω), which is above the 1/f noise corner frequency of the SQUID. The electrical resonance increases energy coupling constant β. The carrier signal at ω_p is precisely balanced by applying feedback forces to the TMs, thus eliminating the oscillator noise at the bridge output.

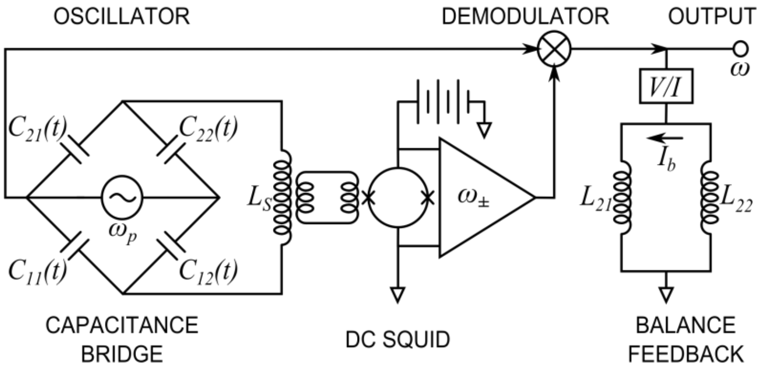

Figure 2. Superconducting tuned capacitor bridge transducer. The bridge output is detected by a dc SQUID. The demodulated signal is fed back to the test masses to keep the bridge precisely balanced.

Sensitivity.—The noise spectral density of a GW detector with an active transducer can be shown to be [5]

$$S_h(f) = \frac{16}{ML^2\omega^4}\left\{\frac{k_B T\omega_D}{Q_D} + \frac{\left|\omega^2-\omega_D^2\right|}{2\omega_p}\left(1+\frac{1}{\beta^2}\right)^{1/2} k_B T_N\right\}, \qquad (1)$$

where M and L are the mass of each TM and the arm-length of the detector; T is the temperature; $\omega_D = 2\pi f_D$ and Q_D are the differential-mode (DM) frequency and Q; and T_N is the noise temperature of the SQUID. In an active transducer, it is possible to achieve $\beta > 1$. For our proposed tuned capacitor bridge transducer, we find [5]

$$\beta = \frac{2CE_p^2 Q_p}{M\left|\omega^2-\omega_D^2\right|}\frac{1}{\sqrt{1+\left(2Q_p\omega/\omega_p\right)^2}}, \qquad (2)$$

where C is the equilibrium capacitance of each sensing capacitor, E_p is the amplitude of the driving electric field at ω_p, and Q_p is the electrical Q.

Table 1 shows the expected $S_h^{1/2}(f)$ for two different sets of detector parameters computed at $f = 1$ Hz. The amplifier noise number is defined by $n \equiv \hbar\omega_p/k_B T_N$. SOGRO is cooled to 1.5 K by pumping on liquid helium or by using cryocoolers, and aSOGRO to 0.1 K by using a He^3/He^4 dilution refrigerator. White noise levels of 120$\hbar$

Parameter	SOGRO	aSOGRO	Method employed (/aSOGRO)
Each test mass M	5 ton	10 ton	Nb square tube
Arm-length L	30 m	100 m	Over "rigid" platform
Antenna temperature T	1.5 K	0.1 K	Superfluid He/dilution refrigerator
Platform temperature T_{pl}	1.5 K	1.5 K	$Q_{pl} = 5 \times 10^6/10^7$
DM frequency f_D	0.01 Hz	0.01 Hz	Magnetic levitation
DM quality factor Q_D	5×10^8	10^9	Surface polished pure Nb
Signal frequency f	0.1-10 Hz	0.1-10 Hz	
Pump frequency f_p	50 kHz	50 kHz	Tuned capacitor bridge transducer
Amplifier noise number n	20	2	Nearly quantum-limited dc SQUID
Detector noise $S_h^{1/2}(f)$	2×10^{-20} Hz$^{-1/2}$	2×10^{-21} Hz$^{-1/2}$	Computed at 1 Hz

TABLE 1. Detector parameters and expected sensitivities of SOGRO and aSOGRO.

at 1.5 K and $10\hbar$ at 0.1 K have been demonstrated by using two-stage dc SQUIDs [8, 9]. The SQUID noise goal for SOGRO and aSOGRO are $20\hbar$ and $2\hbar$, respectively. The parameter values listed in Table 1 lead to $S_h^{1/2}(f) = 1 \times 10^{-20}$ $\mathrm{Hz}^{-1/2}$ and 1×10^{-21} $\mathrm{Hz}^{-1/2}$ for SOGRO and aSOGRO, respectively, at 1 Hz. SOGRO has a very stringent Q requirement. Parasitic forces due to trapped magnetic flux and patch fields may cause damping in the transducer. To minimize such parasitic forces, the TMs need to be constructed with high-purity Nb and with the surface highly polished.

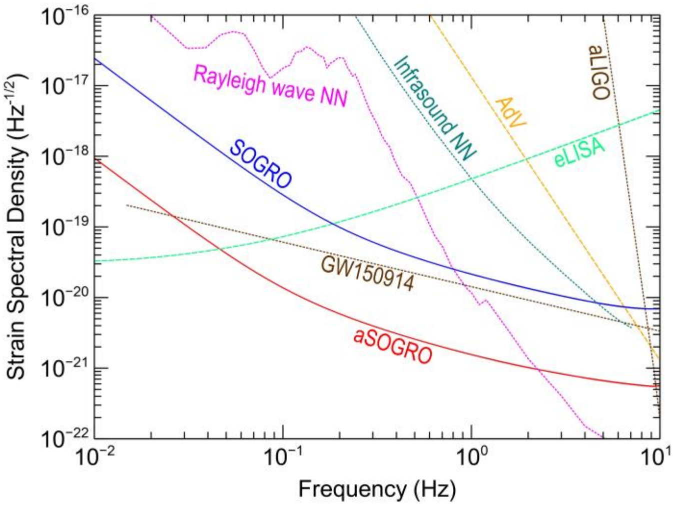

Figure 3. Expected strain sensitivities of SOGRO and aSOGRO. Also plotted is the low-frequency precursor of GW150914. aSOGRO would be sensitive enough to detect BH binaries like GW150915 with a SNR of 10 .

In Fig. 3, we plot the noise spectral densities of SOGRO and aSOGRO, as well as those of the advanced laser interferometers. SOGRO fills the 0.1-10 Hz frequency gap between the terrestrial and future space laser interferometers. Also plotted is the signal spectral density expected from the low-frequency precursor of GW150914 [10]. aSOGRO would be sensitive enough to detect inspiraling stellar mass BH binaries like GW150914 in the frequency band 1-10 Hz with a SNR ~ 10. The signals from such sources last only a fraction of a second in the frequency band of the laser interferometers. In contrast, SOGRO could follow the signal over days or weeks and provide an early warning to the interferometers days before their merger. The NN due to the Rayleigh and infrasound waves are also shown [11, 12]. To reach the detector noise limit, the NN must be mitigated by up to 70 dB for SOGRO and up to 90 dB for aSOGRO.

Seismic noise rejection.—The impact of seismic noise on terrestrial GW detectors at $f \geq 5$ Hz is commonly reduced by designing underground installations [3, 12]. We propose to construct SOGRO underground at depth ~1 km. The seismic noise level of an underground lab at 0.1-10 Hz is $\leq 3 \times 10^{-7}$ $\mathrm{m\,s^{-2}\,Hz^{-1/2}}$ [13], which is 11 and 12 orders of magnitude above the target sensitivity of SOGRO and aSOGRO, respectively. Vibration isolation at such low frequencies is very challenging. SOGRO has a unique capability of *rejecting* the common mode (CM) platform noise to a very high degree. The CM rejection techniques developed for superconducting gravity gradiometers (SGGs) [14, 15] will be applied to SOGRO and improved to one part in 10^{10}. By precisely matching the accelerometer scale factors and further compensating residual errors, the sensitivity to CM acceleration is reduced to 10^{-10}, reducing the isolation requirement to 20 and 40 dB to SOGRO and aSOGRO, respectively.

The platform will be suspended as a long (~50 m) pendulum from its center. The platform will then have horizontal frequencies of ~0.07 Hz and angular frequencies ~1 mHz. This pendulum suspension will provide a passive isolation ≥ 40 dB to horizontal accelerations and 120 dB to angular accelerations at 1 Hz, and much more at 10 Hz. Combined with the CM rejection, the seismic noise will be reduced to below the intrinsic detector noise in all degrees of freedom except for the vertical. For the vertical direction, 40-dB isolation must be provided for aSOGRO by combining passive and active isolation.

NN mitigation.—The NN generated by moving local masses poses a formidable challenge to approaching the detector noise limit at $f \leq 1$ Hz. Both the Rayleigh and infrasound waves are exponentially suppressed with depth z. For the Rayleigh waves (c = 3.5 km/s), there is hardly any attenuation at 0.1 Hz. However, for infrasound waves (c = 330 m/s), the NN is attenuated by up to a factor 6 at 0.1 Hz (depending on its angle of incidence with respect to the surface) and to $< 10^{-8}$ at 1 Hz at 1 km depth. At 0.1 Hz, the NN must be rejected by another three or four orders of magnitude to reach the intrinsic noise limit of SOGRO and aSOGRO, respectively. SOGRO is able to meet this challenge by utilizing the full tensor measurement and simultaneous measurement of CM accelerations with high sensitivity [5].

Conclusions.—A wideband *tensor* GW detector with sensitivity ≤ 10^{-20} $\mathrm{Hz}^{-1/2}$ at 0.1-10 Hz could be constructed by using six widely separated, levitated superconducting TMs. Such a detector would be capable of determining the source direction and wave polarization. The tensor outputs could be combined to better reject the NN caused by the ground motion and atmospheric density fluctuations. Major technical challenges to the new detector are the construction of a large (~100 m), rigid enough (≥ 10 Hz), platform that can be cooled to ≤ 4.2 K and obtaining the required high Q (~10^9) in the levitated superconducting TMs.

With SOGRO, one could set a new limit for GW flux at f = 0.01-10 Hz, not covered by the present terrestrial detectors. Binaries composed of two IMBHs of 10^3-10^4 solar masses at the distance of a few billion light years, and binaries composed of two WDs within the local group would be detected. If the full sensitivity be realized, aSOGRO could detect inspiraling stellar mass BHs like GW150914 and provide an alert to advanced laser interferometers days before merger.

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